Lecture 5: An Introduction to First Order Solution Methods for DSGE Models

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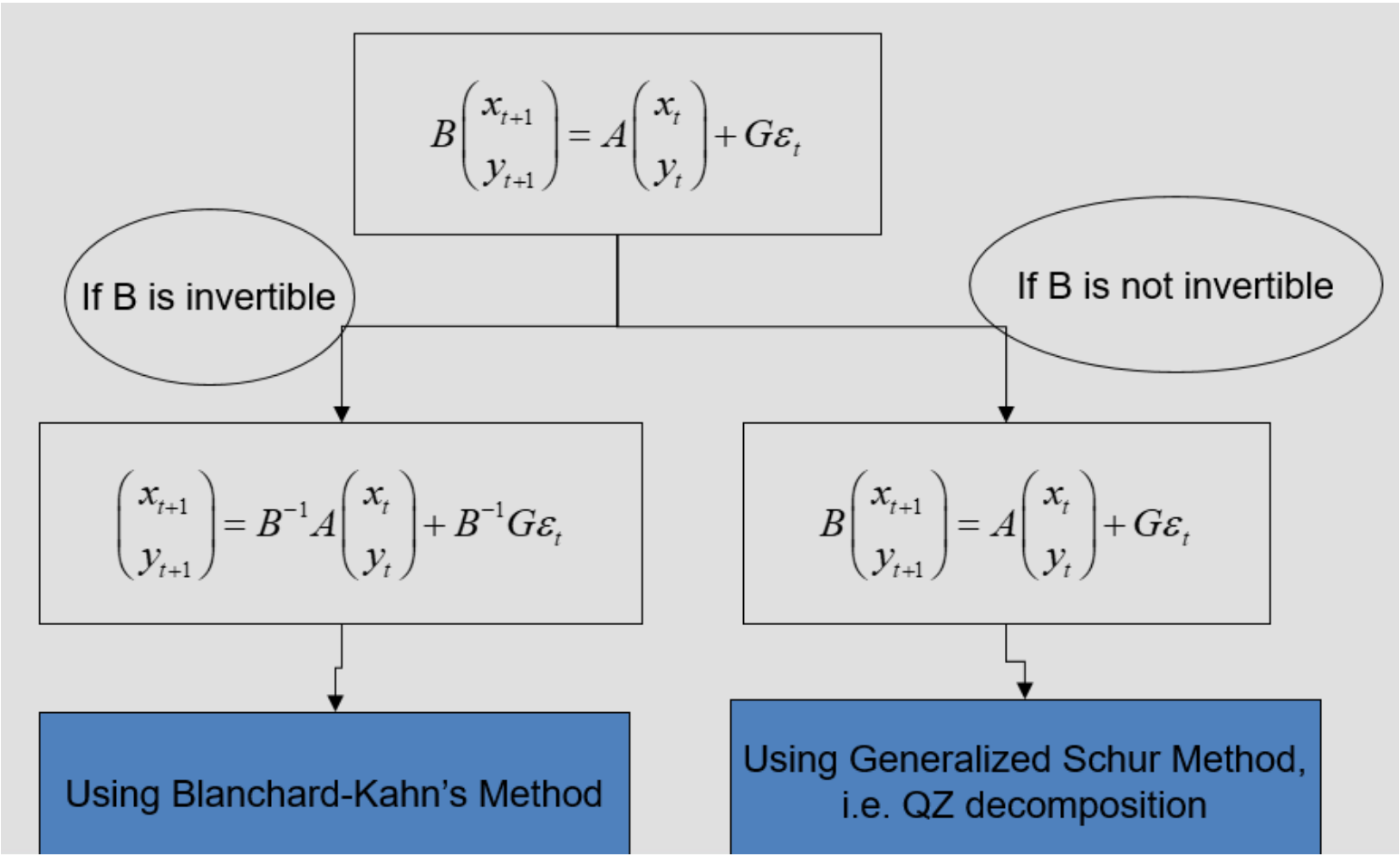
Solution Logic

1. Preparation
   - F.O.C. (First order conditions)
   - #endogenous variables = #equations
     (otherwise Dynare won't work!)
2. Calibration and estimation of parameters
   - Estimation: MLE, GMMM, SMM, Bayesian method
   - Calibration: long-run moments from data
3. Find the steady state
4. Solution: find the policy function
   - What is policy function?
     - express control variables as functions of the state variables
       e.g., express optimal control variables (e.g., consumption and labour) as a function of state variables (e.g., capital and technology shocks) using linear approximation
   - 1st order: log-linearization and 1st order Taylor approximation
   - 2nd order: 2nd order Taylor approximation around steady states
5. Results analysis: various moments (endogenous variables) and IRF

First Order Solution Logic

- Classical 1st order solution methods
  - BK (1980); Blanchard, O.J. & Kahn, C.M. (1980). The solution of linear difference models under rational expectations. *Econometrica*, 48(5), 1305-1311.
  - Schur method (Klein, 2000): Using the generalized Schur form to solve a multivariate linear rational expectations model. *Journal of Economic Dynamics and Control*, 24(10), 1405-1423.
  - Uhlig (1999) undetermined coefficients method: A toolkit for analyzing nonlinear dynamic stochastic models easily
- McCandless, G. (2008). The ABCs of RBCs: An Introduction to Dynamic Macroeconomic Models, Havard University Press.
  - not difficult to understand
  - especially useful to learn RBC models

BK and Schur Methods (1 0f 2)



BK and Schur Methods (2 0f 2)

- x_t: predetermined variables which were determined in last period (i.e., state variables)
  - e.g., stock of capital
- y_t: non-predetermined variables which are determined at current period (i.e., control variables)
  - e.g, consumption
- Key difference: predetermined variable vs. non-predetermined variable:
  - values of predetermined variables at time t do not depend on the values of time t shocks
  - while non-predetermined variables do

Uhlig Method (1 0f 2)

Problem:

$$\begin{pmatrix} 0 = Ax_t + Bx_{t-1} + Cy_t + Dz_{t-1} \\ 0 = E_t\left(Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right) \\ z_{t+1} = Nz_t + \varepsilon_{t+1}, E_t\left(\varepsilon_{t+1}\right) = 0. \end{pmatrix}$$

Solution:

$$\begin{pmatrix} x_t = Px_{t-1} + Qz_t \\ y_t = Rx_{t-1} + Sz_t. \end{pmatrix}$$

Note:

x_t is the endogenous state variables;
y_t is the endogenous control variables,
sometime called jump variables;

Uhlig Method (2 0f 2)

- z_{t+1}: exogenous shock
- substitute x_t and y_t into original equations
- then find P, Q, R and S

Summary

- BK(1980) and Schur methods
  - smaller application scope
  - simple and easy to understand
  - need to be very familiar with program tools (e.g., Matlab)
- Uhlig(1999) method
  - larger application scope
  - undetermined coefficients method
  - a standby package could be used which was provided by the author
- Dynare
  - find a solution is much easier than before
  - can only focus on the model itself instead of solving technical details